Electrostatic instabilities in current-carrying magnetoplasmas with equilibrium density and ion velocity gradients

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Detailed studies of electrostatic wave instabilities in a current-carrying magnetoplasma with equilibrium density and ion velocity gradients are presented. For this purpose, a general dispersion relation is derived by using a non-Boltzmann electron response as well as an ion density perturbation, which includes the ion-neutral drag. Our dispersion relation contains previously known results as limiting cases, but it also includes some additional instabilities associated with the electron-wave resonant interaction. The present results can help to understand the origin of the nonthermal electrostatic waves in laboratory and space plasmas where there are free energy sources due to ion velocity gradients and streaming particle motions.

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In the 1960s D'Angelo [1] predicted a novel electrostatic instability in the presence of a parallel (to the external magnetic field $\hat{\mathbf{z}}B_0$) ion velocity gradient ($\partial u_{i0}/\partial x = u_{i0}'$), where $\hat{\mathbf{z}}$ is the unit vector along the z axis, B_0 is the strength of the magnetic field, and u_{i0} is the magnetic field aligned unperturbed ion flow speed. The physical mechanism of the instability is attributed to an adverse phase lag between the parallel ion velocity perturbation and the wave potential ϕ due to the $\mathbf{E}\times\hat{\mathbf{z}}$ convection of the equilibrium ion flow speed, where $\mathbf{E}=-\nabla\phi$ is the wave electric field. The parallel ion velocity gradient instability also exists in collisional [2] and nonuniform [3,4] magnetoplasmas.

Recently, there has been a great deal of interest in studying experimentally [2,5–9] and theoretically [10,11] the excitation of electrostatic waves by the parallel ion velocity gradient. Koepke [12] presented contributions of Q-machine experiments to the understanding of auroral wave activities and associated particle acceleration processes. However, it seems that the combined effects of the sheared ion flows, particle streamings, and interparticle collisions have not yet been fully investigated. Accordingly, in this Brief Report, we present an improved study of electrostatic wave instabilities in current-carrying collisional magnetoplasmas with equilibrium density and ion velocity gradients, by using a non-Boltzmann electron response and a hydrodynamic description for the ions. We assume that the sheared ion flows and inhomogeneities are maintained by external sources, the details of which are not essential here.

We consider a multicomponent plasma whose constituents are electrons, ions, high-Z impurities (e.g., charged dust particles [13]), and neutrals. The plasma has an equilibrium density gradient ($\partial n_{i0}/\partial x = n_{i0}'$) along the x axis. The unperturbed ion number density is $n_{i0}(x)=n_{e0}+Z_d n_{d0}$, where n_{e0} and n_{d0} are the unperturbed electron and dust number densities, respectively, and Z_d is the number of electrons residing on each dust grain. The latter is supposed to be stationary, since the time scales of our interest are shorter than the dust plasma period.

In the presence of low-frequency (in comparison with the electron gyrofrequency $\omega_{ce}=eB_0/m_e c$, where e is the magnitude of the electron charge, m_e is the electron mass, and c is the speed of light in vacuum) long-wavelength (in comparison with the electron and ion thermal gyroradii) electrostatic perturbations with $|\omega - k_z u_{e0} - k_y u_E - \omega_{e*} + i\nu_{en}| \ll k_z V_{Te}$ and $\nu_{en} \ll k_z V_{Te}$, we have for the electron number density perturbation [13],

$$n_{e1} \approx n_{e0}\frac{e\phi}{T_e}\left(1 + i\sqrt{\frac{\pi}{2}}\frac{\Omega_e}{k_z V_{Te}}\right), \tag{1}$$

where $\Omega_e = \omega - k_z u_{e0} - k_y u_E - \omega_{e*}$, u_{e0} is the magnetic field aligned unperturbed electron streaming speed, $u_E = cE_{0x}/B_0$ is the cross-field drift due to a dc electric field $-\hat{\mathbf{x}}E_{0x}$, T_e is the electron temperature, ω is the wave frequency, $\omega_{e*} = -k_y(cT_e/eB_0 n_{e0})\partial n_{e0}/\partial x$ is the electron drift wave frequency, ν_{en} is the electron-neutral collision frequency, k_z is the magnetic field aligned wave number, and $V_{Te}=(T_e/m_e)^{1/2}$ is the electron thermal speed.

The ion number density perturbation n_{i1} is determined from

$$\partial_t n_{i1} + \nabla\cdot(n_{i0}\mathbf{v}_i) = 0 \tag{2}$$

and

$$(\partial_t + \mathbf{v}_i\cdot\nabla + \nu_{in})\mathbf{v}_i = -\frac{e}{m_i}\nabla\left(\phi + \frac{\gamma_i T_i}{e n_{i0}} n_{i1}\right) + \omega_{ci}\mathbf{v}_i\times\hat{\mathbf{z}} - \frac{1}{m_i n_{i0}}\nabla\cdot\mathbf{\Pi}_i, \tag{3}$$

where $\mathbf{v}_i$ is the ion fluid velocity, ν_{in} is the ion-neutral collision frequency, m_i is the ion mass, γ_i is the adiabatic index

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for the ion fluid, T_i is the ion temperature, $\omega_{ci}=eB_0/m_ic$ is the ion gyrofrequency, and $\mathbf{\Pi}_i$ is the ion stress tensor.

Equations (1)–(3) are closed by means of Poisson's equation

$$\nabla^2\phi=4\pi e(n_{e1}-n_{i1}). \tag{4}$$

Letting $\mathbf{v}_i=\hat{\mathbf{z}}(u_{i0}+v_{iz})+\mathbf{v}_{i\perp}$, where v_{iz} and $\mathbf{v}_{i\perp}$ are the parallel and perpendicular components of the ion fluid velocity perturbation, we obtain from Eq. (3),

$$(D_t^2+\omega_{ci}^2)\mathbf{v}_{i\perp}=-\frac{e}{m_i}D_t\nabla_\perp\varphi+\frac{e}{m_i}\omega_{ci}\hat{\mathbf{z}}\times\nabla\varphi \tag{5}$$

and

$$D_tv_{iz}+\mathbf{v}_{i\perp}\cdot\nabla u_{i0}=-\frac{e}{m_i}\partial_z\varphi-\frac{1}{m_in_{i0}}(\nabla\cdot\mathbf{\Pi}_i)_z, \tag{6}$$

where $D_t=\partial_t+u_{i0}\partial_z+\nu_{in}\equiv d_t+i\nu_{in}$ and $\varphi=\phi+\gamma_iT_in_{i1}/en_{i0}$.

Inserting Eqs. (5) and (6) into Eq. (2) we obtain after some manipulation

$$\begin{aligned}&[(D_t^2+\omega_{ci}^2)(D_td_t-\gamma_iV_{Ti}^2\partial_z^2)-D_t^2\gamma_iV_{Ti}^2\nabla_\perp^2]n_{i1}-\frac{e}{m_i}\omega_{ci}\hat{\mathbf{z}}\\&\quad\times\nabla n_{i0}\cdot\nabla D_t\phi-\frac{n_{i0}e}{m_i}(D_t^2\nabla^2+\omega_{ci}^2\partial_z^2)\phi+\frac{n_{i0}e}{m_i}\omega_{ci}\hat{\mathbf{z}}\\&\quad\times\nabla u_{i0}\cdot\nabla\partial_z\phi=0,\end{aligned} \tag{7}$$

where $V_{Ti}=(T_i/m_i)^{1/2}$ is the ion thermal speed. Supposing that n_{i1} and ϕ are proportional to $\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)$, where $\mathbf{k}=\mathbf{k}_\perp+\hat{\mathbf{z}}k_z$ is the wave vector, we Fourier analyze Eq. (7) and obtain

$$\begin{aligned}&\{[(\Omega+i\nu_{in})^2-\omega_{ci}^2][\Omega(\Omega+i\nu_{in})-\gamma_ik_z^2V_{Ti}^2]\\&\quad-(\Omega+i\nu_{in})^2\gamma_ik_\perp^2V_{Ti}^2\}\frac{n_{i1}}{n_{i0}}\\&\quad=\frac{e}{m_i}\bigg[(\Omega+i\nu_{in})\omega_{ci}\mathbf{k}\cdot(\hat{\mathbf{z}}\times\nabla\ln n_{i0})\\&\quad-k_z^2\omega_{ci}^2\left(1-\frac{\mathbf{k}\cdot(\hat{\mathbf{z}}\times\nabla u_{i0})}{k_z\omega_{ci}}\right)+k^2(\Omega+i\nu_{in})^2\bigg]\phi,\end{aligned} \tag{8}$$

where $\Omega=\omega-k_zu_{i0}$ and $k^2=k_\perp^2+k_z^2$.

Inserting Eqs. (1) and (8) into the Fourier transformed version of Eq. (4), we obtain for $k^2\lambda_{De}^2\ll 1$, where λ_{De} is the electron Debye radius, the dispersion relation

$$\begin{aligned}&\left(1+i\sqrt{\frac{\pi}{2}}\frac{\Omega_e}{k_zV_{Te}}\right)\{[(\Omega+i\nu_{in})^2-\omega_{ci}^2]\\&\quad\times[\Omega(\Omega+i\nu_{in})-\gamma_ik_z^2V_{Ti}^2]-(\Omega+i\nu_{in})^2\gamma_ik_\perp^2V_{Ti}^2\}\\&\quad=C_s^2\bigg[(\Omega+i\nu_{in})\omega_{ci}\mathbf{k}\cdot(\hat{\mathbf{z}}\times\nabla\ln n_{i0})\\&\quad-k_z^2\omega_{ci}^2\left(1-\frac{\mathbf{k}\cdot(\hat{\mathbf{z}}\times\nabla u_{i0})}{k_z\omega_{ci}}\right)+k^2(\Omega+i\nu_{in})^2\bigg],\end{aligned} \tag{9}$$

where $C_s=(n_{i0}T_e/n_{e0}m_i)^{1/2}$ is the dust ion-acoustic speed. Equation (9) is the general dispersion relation for low-frequency electrostatic waves in nonuniform current-carrying collisional magnetoplasmas, with equilibrium density and parallel ion velocity gradients. It can be numerically investigated to understand the interplay between the electron streamings, ion velocity gradients, and ion-neutral collisions.

Several comments are in order. First, for $\nu_{in},kV_{Ti}\ll|\Omega|\ll\omega_{ci}$, and $k_z\ll k_y$, we obtain from Eq. (9),

$$\begin{aligned}&\Omega^2-\Omega\omega_{i*}-\omega_a^2\left(1-\frac{k_y}{k_z}S_v\right)\\&\quad+i\sqrt{\frac{\pi}{2}}\frac{\omega-k_zu_{e0}-k_yu_E-\omega_{e*}}{k_zV_{Te}}\frac{\Omega^2}{1+k_\perp^2\rho_s^2}=0,\end{aligned} \tag{10}$$

which is the dispersion relation for the coupled dust ion-acoustic-drift (DIAD) waves including the electron-Landau contribution [the last term in the right-hand side of Eq. (10)]. Here, we have denoted $\omega_{i*}=-k_yC_s^2n_{i0}'/\omega_{ci}n_{i0}(1+k_\perp^2\rho_s^2)$, $\rho_s=C_s/\omega_{ci}$, $\omega_a=k_zC_s/(1+k_\perp^2\rho_s^2)^{1/2}$, and $S_v=u_{i0}'/\omega_{ci}$. For $S_v<0$, Eq. (10) exhibits the growth of the coupled DIAD waves when $k_zU_0>\omega_r$, where $U_0=u_{e0}+u_{i0}$ and $\omega_r=k_zu_{i0}+\omega_{e*}+(\omega_{i*}/2)\pm(1/2)[\omega_{i*}^2+4\omega_a^2(1+k_y|S_v|/k_z)]^{1/2}$. On the other hand, for $S_v>k_z/k_y$ and $\omega_a^2k_yS_v/k_z>\omega_{i*}^2/4$, we have an oscillatory instability [3] of the driftlike waves without the inverse Landau damping effect. In a plasma with a flat density profile and without the electron Landau damping (growth) effect, we have the shear flow instability growth rate

$$\gamma_s=k_zC_s\left(\frac{k_y}{k_z}S_v-1\right)^{1/2}, \tag{11}$$

which attains a peak value $\gamma_p=k_yC_sS_v/2$ for $k_z=k_yS_v/2$.

Second, we focus on $|\Omega+i\nu_{in}|\ll\omega_{ci}$ and $|\Omega|\ll\nu_{in}$. Here, Eq. (9) reduces to

$$\begin{aligned}&\left(1+i\sqrt{\frac{\pi}{2}}\frac{\omega-k_zu_{e0}-k_yu_E-\omega_{e*}}{k_zV_{Te}}\right)\Omega\\&\quad=-i\frac{k_z^2C_s^2}{\nu_{in}}\left(1-\frac{k_y}{k_z}S_v\right)-\frac{C_s^2}{\omega_{ci}}k_y\frac{n_{i0}'}{n_{i0}}-i\nu_{in}k_\perp^2\rho_s^2.\end{aligned} \tag{12}$$

Letting $\omega=\omega_r+i\omega_i$ in Eq. (12), where $\omega_i(<\omega_r)$ is the growth rate, we obtain $\omega_r=k_z u_{i0}+\omega_{e*}+\omega_{i*}\equiv k_z u_{i0}+\omega_*$, $k_z U_0+k_y u_E>\omega_*$, and the growth rate

$$\omega_i=\omega_*\frac{k_z U_0+k_y u_E}{k_z V_{Te}}+\frac{k_z k_y C_s^2}{\nu_{in}}S_v>\frac{k_z^2 C_s^2}{\nu_{in}}+\nu_{in}k_\perp^2\rho_s^2 \tag{13}$$

for $S_v>k_z/k_y$. Equation (13) shows that energy due to particle streaming and ion velocity gradients is coupled to driftlike oscillations in a nonuniform collisional magnetoplasma.

Third, we consider the coupled ion-cyclotron-drift-dust acoustic modes in a collisionless magnetoplasma with cold ions. Here, Eq. (9) gives

$$1+i\sqrt{\frac{\pi}{2}}\frac{\omega-k_z u_{e0}-k_y u_E-\omega_{e*}}{k_z V_{Te}} -\frac{k_\perp^2 C_s^2}{\Omega^2-\omega_{ci}^2}-\frac{k_z^2 C_s^2}{\Omega^2}\left(1-\frac{k_y}{k_z}S_v\right)-\frac{k_y C_s^2\omega_{ci}}{\Omega(\Omega^2-\omega_{ci}^2)}\frac{n_{i0}'}{n_{i0}}=0, \tag{14}$$

which (for $k_z\ll k_y$) generalizes the work of Shukla and Stenflo [4] by including the electron-Landau contribution. In the absence of the latter, Eq. (14) yields

$$\Omega^2=\tfrac{1}{2}(\omega_{ci}^2+k_\perp^2C_s^2+k_z^2V_s^2)\pm\tfrac{1}{2}[(\omega_{ci}^2+k_\perp^2C_s^2+k_z^2V_s^2)^2 -4k_z^2V_s^2\omega_{ci}^2]^{1/2}, \tag{15}$$

which depicts an oscillatory instability of the electrostatic ion-cyclotron waves, provided that k_z/k_y and S_v are sufficiently large. Here, we have denoted $V_s^2=C_s^2(1-k_yS_v/k_z)$. The velocity shear origin of low-frequency electrostatic ion-gyroresonant waves in a nonuniform space plasma, which are described by Eq. (14), has been reported by Carroll *et al.* [14].

To summarize, we have presented a unified picture of some instabilities involving low-frequency long-wavelength electrostatic modes in a current-carrying collisional magnetoplasma with equilibrium density and parallel ion velocity gradients. We have used a non-Boltzmann electron response arising from the electron-wave interaction, and have derived a general dispersion relation by employing hydrodynamic equations for the ions and the Poisson equation. The general dispersion relation exhibits the interplay between the ion velocity gradient and the inverse electron-Landau damping effect. It is found that the latter causes instability of the coupled drift-dust ion-acoustic waves when $u_{i0}'<0$. In addition, when $u_{i0}'>0$, the driftlike waves in collisionless and collisional magnetoplasmas are destabilized due to the combined effects of the ion velocity gradient and streaming particle motions. Furthermore, it is also found that the parallel ion velocity gradient can drive electrostatic ion-cyclotron waves. The present investigation should therefore be useful in understanding the threshold behavior and the increment of broadband electrostatic waves that are spontaneously excited in current-carrying magnetoplasmas (viz., low-temperature laboratory and Earth's ionospheric plasmas) which contain sheared ion flows, equilibrium density gradients, and high-Z charged impurities (dust grains). A rapid cross-field enhanced ion transport in a plasma with sheared parallel flow is also expected [15].

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